Multi-Constraint Stock Portfolio Optimization for Retail Investors Using Branch and Bound Algorithm

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Abstract*— Retail investors often deal with challenges that aren't always considered in traditional portfolio optimization models like tight budgets, flat trading fees, and limits on how many stocks they can realistically manage. Most standard models assume perfect market conditions and allow for smooth, continuous decisions, which doesn't reflect the real world. In this paper, we introduce a more practical, constraint-aware approach to portfolio building using the Branch and Bound algorithm. This method helps handle situations where investors can only take long positions, need to stay within strict budget constraints, pay both fixed and variable transaction fees, and can only hold a limited number of different assets. These types of constraints create a complex problem that standard convex optimization techniques can't solve well. Our approach embraces that complexity using discrete optimization, which is better suited for the kinds of decisions retail investors face. We test the method using real historical stock data, and our results show that it can generate portfolios that balance returns and risk effectively while also being realistic, affordable, and actionable for everyday investors.

Keywords*— portfolio optimization, retail investors, transaction costs, asset limits, Branch and Bound, discrete decision-making.

I. Introduction

In today's interconnected world, nearly every move we make ties back to economics. And when it comes to investing, the age-old quest for the perfect balance between risk and reward through smart asset allocation is still what we are looking for. This concept is not merely theoretical; it is widely applied in practical financial decision-making. For us, as individuals or retail investors navigating today's ever-changing markets, the core mission is to build and fine tune our investments portfolios to hit our personal financial goals.

At its heart, optimizing a portfolio means figuring out the best way to invest your money to get the most returns over time, all while staying within your comfort zone for risk. It's a classic two-sided challenge, always juggling conflicting aims. A key strategy here is diversification, like spreading your bets so that if one investment stumbles, the others can rise and keep your overall portfolio in the plus side.

The journey of portfolio optimization really took off with Markowitz's groundbreaking work in 1952, laying the foundation for how we think about investing quantitatively. Since then, the field has exploded, thanks to incredible leaps in software development, a massive surge in computing power, and the use of parallel processing. This evolution marks a pivotal shift that we're no longer stuck with oversimplified, purely theoretical models. Instead, we can now tackle the messy, real-world complexities of financial markets with powerful, computation-heavy approaches. The rise of multi-criteria optimization also plays a huge role in broadening the scope beyond just money, letting us factor in things like a company's impact on the environment or its ethical practices. This holistic view is incredibly relevant for many retail investors today who want their investments to align with their values, leading to more personal and comprehensive ways to build wealth.

However, the real puzzle and the reason we need algorithms comes when we try to solve real-world limitations in these sophisticated models. Think about it when you can't buy half a share, there are always transaction fees, you might want to spread your money across different sectors, or there could be a minimum amount you need to invest. These aren't minor details, they're essential for creating investment strategies that are robust, complete, and most importantly, doable. If we ignore them, our optimal plans on paper can quickly become impossible to manage, too expensive to execute, or just unrealistic. Many traditional retail investor strategies often found in academic literature simplify things by skipping these non-partial constraints and fixed fees, largely for mathematical convenience. While this might be acceptable for giant institutional portfolios, these shortcuts really start breaking down when we're talking about the wealth levels of young or novice investors. This starkly underlines why these often-overlooked practical constraints are so critically important for the average investor.

Simplifying investment models by ignoring real-world constraints leads to a significant disconnect, especially for retail investors. A giant financial institution managing billions probably won't even notice a small $5 trading fee or the rule to buy shares in blocks of 100, it's a tiny drop in their ocean. But for you, the individual investor with a smaller portfolio, these frictional costs (whether it's a flat fee every time you trade or having to buy full shares) can quickly add up. They become a significant drag, eating away at your potential profits. This means a portfolio that looks perfect on paper might be inefficient or even impossible to put into practice. So, while it makes our models more complicated, carefully including these real-world limitations isn't just for academics. It's an absolute

must if we want to give retail investors investment strategies that are genuinely useful, realistic, profitable, and specifically designed for them. To address these discrete and non-convex challenges, this paper adopts the Branch and Bound algorithm, a combinatorial optimization technique well-suited for handling integer decisions and constraints. This method enables practical portfolio construction tailored to the specific needs of retail investors.

II. THEORY FOUNDATION

A. *Investment Objectives*

Essentially, investors want to get the best possible return on their money, but only if the risk stays within a level, they're comfortable with – that's what we mean by 'risk appetite.' We can also flip that around and say they want to minimize risk while still aiming for a certain minimum return or simply find the sweet spot using a 'utility function' that balances both. In practice, though, it's often easier for investors to think about directly limiting their portfolio's risk (like its ups and downs, or 'variance') rather than setting a minimum profit goal. On top of that, advanced strategies now also include something called Conditional Value-at-Risk (CVaR) to help manage 'tail risk,' which is super important for protecting against those really big, unexpected losses

B. *Key Constraints for Retail Investors*

These constraints are essential features that shape the portfolio optimization problem to better reflect real-world investment conditions.

1. **Budget and Holding Limits:**

   - **Long-Only or No-Short-Selling Constraint**: This is a common constraint that dictates that investors can only purchase stocks, taking long positions, meaning the allocation weight for each asset must be non-negative ($w \geq 0$). This is a linear and convex constraint.

   - **Capital Budget Constraint**: Assuming no short-shelling and no other forms of leverage, the total portfolio must satisfy a budget constraint, typically

$$1^T w + c = 1 \qquad (1)$$

   where w is a vector of portfolio weights, where each element represents the proportion of your total portfolio invested in a specific risky asset. Prepare Your Paper Before Styling. 1 is a vector of one, T (superscript) denotes the transpose operation, turning one into a row vector. So, 1^T w calculates the sum of all individual asset weights in w, c represents the proportion of your total portfolio held as cash.

   - **Holding Constraints**: Practitioners set limits on maximum positions ($l \leq w \leq u$) to prevent overexposure and ensure diversification. Minimum positions can also be set if certain assets are desired. These are linear and convex constraints.

2. **Transaction Costs:**

Transaction costs (e.g., commissions, fees) reduce the funds' capital and must be properly to avoid substantial impacts on portfolio performance.

Realistic transaction costs are often non-convex. They typically include a fixed fee (η) plus a variable component (ρ * Value), meaning the transaction cost per unit is higher for smaller amounts. This results in a non-convex function, as the transaction cost decreases relatively when the trading amount increases. This non-convexity makes the optimization problem significantly more challenging.

The moment you introduce fixed trading fees or more complex concave transaction costs; the entire portfolio optimization problem changes dramatically. Unlike simpler, more straightforward situations that standard solvers (like LP (Linear Programming) / QP (Quadratic Programming)) can easily manage, these kinds of costs make the problem non-convex. This means a solver might get stuck on a local optimum, a satisfactory solution that is not the absolute best one globally. For individual investors, this is not just theory; it has real financial consequences. If you are making lots of small trade, which is common for lots of individual investors, those fixed fees can eat up a huge chunk of your potential profits, making the trades surprisingly expensive. Any investment model that ignores this reality would suggest trades that lose you money. This is exactly why we need sophisticated tools like the Branch and Bound algorithm. It is specifically designed to systematically search for the absolute best solution, even when these tricky, non-convex costs are involved, by wisely introducing integer variables into the problem.

3. **Cardinality Constraints**

Cardinality constraints limit the number of assets that can be included in a portfolio to a predefined number K. This constraint is particularly relevant for retail investors, who may not have the capacity financially or administratively to manage many assets simultaneously.

To model this, we introduce binary decision variables y_i for each asset iii, where:

$$y_i = \begin{cases} 1, & \text{if asset } i \text{ is included in the portfolio} \\ 0, & \text{otherwise} \end{cases}$$

These binary variables are then used to impose the cardinality constraint:

$$\sum_{i=1}^{n} y_i \leq K \qquad (2)$$

where:

- n is the total number of candidate assets, and
- K is the maximum number of assets that can be selected

To ensure consistency between the binary decision y_i and the actual allocation w_i (i.e., the fraction of capital invested in asset iii), the following coupling constraint is added:

$$w_i \leq y_i \cdot u_i, \quad \forall i = 1, 2, ..., n \tag{3}$$

where:

- w_i is a continuous variable representing the fraction of budget allocated to asset i,
- u_i is upper bound (maximum allowable allocation) for asset i,
- if $y_i = 0$, then w_i must be 0 (i.e., the asset is not included),
- if $y_i = 1$, then w_i can be any value up to u_i.

This transformation adds further combinatorial complexity to the portfolio optimization problem, converting it into a Mixed-Integer Linear Programming (MILP) or Mixed-Integer Quadratic Programming (MIQP) model depending on the chosen risk metric. The inclusion of integer and binary variables, along with upper-bound continuous weights, results in a highly discrete search space. These characteristics make Branch and Bound an ideal algorithmic approach, as it systematically explores feasible combinations while pruning suboptimal ones efficiently. In this way, the optimization process can realistically account for retail investors' practical limitations such as discrete purchases and portfolio simplicity while still aiming for robust financial performance.

4. **Minimal Transaction Unit (MTU) Constraints**

Retail brokers often enforce trading in minimum units, such as lots of one hundred shares. To respect this, we model the weight of each asset w_i as multiple of its minimal tradable unit m_i, with an integer variable z_i:

$$w_i = z_i \cdot m_i, \quad z_i \in \mathbb{Z}_{\geq 0} \tag{4}$$

This further increases the discrete nature of the problem and transforms it into a Mixed-Integer Nonlinear Programming (MINLP) problem. This constraint is particularly relevant for low-budget investors for whom fractional shares are not an option.

5. **Diversification and Sector Constraints**

To prevent risk concentration, diversification constraints can be applied. One approach is to restrict the portfolio's spread using squared ℓ_2-norm:

$$\|\mathbf{w}\|_2^2 \leq D \tag{5}$$

Smaller D promotes more evenly distributed allocations. Additionally, sector-based diversification constraints can be imposed:

$$\sum_{i \in \mathcal{S}_j} w_i \leq S_j, \quad \forall j \tag{6}$$

where S_j is the set of assets in sector j, and S_j is the maximum allowable weight for that sector. These constraints help manage exposure to sector-specific risks.

6. **Turnover Constraints**

In multi-period or rebalancing scenarios, minimizing turnover is essential to avoid excessive trading costs. Turnover is measured as the ℓ_1-norm of the difference between the new portfolio w and the current portfolio w_0:

C. Mathematical Nature of Constraints and Non-Convexity Challenges

Many commonly used portfolio constraints such as budget limits, long-only positions, holding bounds, and turnover limits are either linear or convex. As a result, they can be efficiently handled using conventional convex optimization techniques. However, when constraints like cardinality (limiting the number of assets in the portfolio) and realistic transaction costs (such as fixed fees or concave cost functions) are introduced, the problem becomes non-convex. This transforms the optimization task into a Mixed-Integer Linear Program (MILP) or even a Mixed-Integer Nonlinear Program (MINLP), both of which are significantly more difficult to solve than their convex counterparts.

Classical models like Markowitz's mean-variance optimization assume continuous asset weights, leading to well-behaved convex problems that can be solved using Quadratic Programming (QP). However, introducing real-world constraints such as the need to decide whether a stock is included at all (cardinality) or to account for fixed transaction fees requires binary or integer decisions (e.g., "Do I include this stock?" or "How many full lots should I buy?"). These decisions shift the problem into the realm of Mixed-Integer Programming (MIP), where standard continuous solvers are no longer applicable. This fundamental change in structure is precisely why Branch and Bound algorithms are essential; they are purpose-built to efficiently explore discrete solution spaces and find globally optimal solutions under such complex constraints.Table 1 below summarizes key constraints relevant to retail investors and their computational implications:

Constraint Name	Purpose	Mathematical Formulation	Complexity
Budget	Ensures total capital is fully allocated	$\Sigma x_i = 1$	Convex
Long-Only	Prevents short shelling; only allows asset purchases	$x_i \geq 0$	Convex
Holding Limits	Restricts asset concentration; promotes diversification	$l_i \leq x_i \leq u_i$	Convex
Cardinality	Limits number of active holdings and manageability	$MinNum \leq \Sigma y_i \leq MaxNum$	Non-Convex (requires binary Vars)
Fixed Transaction Costs	Models fixed trading fees regardless of trade size	$Cost = fixed\_fee \times z_i$	Non-Convex (binary decision: trade/no trade)
Minimal Transaction Unit	Enforces trading in minimum lot sizes, e.g., whole shares	$x_i = 0$ or $x_i \geq min\_unit\_value_i$	Non-Convex (integer or binary vars)
Portfolio Turnover	Limits amount of rebalancing (change from current holdings)	$\|x - x_0\|_1 \leq \tau$	Convex (L_1 norm)

Table 1. Key Portfolio Constraints for Retail Investors

D. Branch and Bound

First, let's understand what Branch and Bound is and how does it work? The Branch and Bound (B&B) algorithm is a powerful and versatile general-purpose algorithmic framework specifically designed for solving optimization problems that involve discrete or integer decisions. Unlike exhaustive enumeration, which checks every single possibility (often computationally infeasible), B&B systematically searches for the optimal solution without having to explore the entire solution space. It is particularly effective for problems where the objective is to either minimize or maximize a function, subject to various constraints.

B&B operates on the principle of dynamically building a state-space tree to explore potential solutions. While it shares similarities with tree-search algorithms like Depth-First Search (DFS) or Breadth-First Search (BFS) in its exploration method, its core strength lies in its intelligent pruning mechanism that significantly reduces the search space. B&B can be seen as a sophisticated combination of BFS and a least cost search strategy. Instead of simply expanding nodes based on their generation order (like FIFO in pure BFS), B&B prioritizes expanding the node that has the most promising cos" (or bound).

At an elevated level, the algorithm systematically works through the following iterative steps:

1. Relaxing the Integrality Constraints (Bounding Phase)

   - The first crucial step in B&B is to relax the complex integer or discrete constraints of the original optimization problem. For instance, if a problem requires variables to be whole numbers (e.g., you can only buy 5 or 6 shares, not 5.5), these constraints are temporarily ignored, allowing the variables to take on continuous (fractional) values.
   - This transformed problem is often a Linear Program (LP) if the original problem becomes linear after relaxation, or a Quadratic Program (QP) if the objective function is quadratic (like variance in portfolio optimization) and constraints are linear.
   - Solving this relaxed, continuous problem provides an optimistic bound on the objective function. For a minimization problem, this relaxed solution gives a lower bound (the best you could possibly do if you didn't have to stick to integers), and for a maximization problem, it gives an upper bound. This bound is critical because it tells us the best possible outcome within that specific branch of the search tree.

2. Branching on a Fractional Decision Variable

If the solution obtained from the relaxed problem contains any variables that are supposed to be integers but end up with fractional values (e.g., suggesting you buy 2.7 shares of stock), the algorithm performs a branching operation.

This involves creating two (or more) new subproblems from the current node. For a fractional variable x_i with a value of v, one subproblem will add the constraint $x\_i \leq floor(v)$ (rounding down), and the other will add $x\_i \geq ceil(v)$ (rounding up). For example, if $x\_i = 2.7$, one branch explores solutions where $x\_i \leq 2$, and the other where $x\_i \geq 3$.

This process effectively divides the original problem's feasible region into smaller, mutually exclusive sub-regions.

3. Bounding Each Subproblem

Each newly created subproblem is then solved by relaxing its integrality constraints, just like the initial problem. The optimal objective value from this relaxed version serves as the bound for that specific subproblem. This bound indicates the best possible result that could be achieved if we were to continue exploring solutions within that branch. The cost $\hat{c}(i)$ of a node i typically estimates the cheapest path to a goal node through i. In a general scenario

where the solution's location is unknown, ĉ(i) is estimated heuristically and represents a lower bound on the search cost from state i. It can be seen as ĉ(i) = f(i) + ĝ(i), where f(i) is the cost to reach node i from the root, and ĝ(i) is the estimated cost from node i to the goal node.

4. Pruning (Eliminating Unpromising Branches)

This is where Branch and Bound gain their efficiency. The algorithm prunes (discards) any branch of the search tree that is deemed unpromising, meaning it cannot lead to a better solution than the best feasible (integer) solution found so far (known as the incumbent solution). A branch can be pruned if:

- Its LP/QP relaxation is infeasible (no solution exists that satisfies all constraints, even relaxed ones).
- Its calculated bound is worse than the objective value of the current incumbent integer solution. For example, in a minimization problem, if a branch's lower bound is already higher than an integer solution found elsewhere, that branch can be cut.
- The LP/QP relaxation of a subproblem yields an integer solution directly. If this integer solution is better than the current incumbent, it becomes the new incumbent solution.

5. Updating the Incumbent Solution

Whenever a relaxed subproblem yields a solution where all integer variables happen to be integers, and this solution is better than the current best integer solution found so far, it becomes the new incumbent solution. This incumbent solution is the best feasible (integer-respecting) solution known at any given point.

6. Repeating the Process

The algorithm continually selects the most promising live node (the one with the best bound, e.g., the smallest lower bound for a minimization problem) and repeats the branching, bounding, and pruning steps.

This iterative process continues until there are no more active branches that could potentially yield a better solution than the current incumbent. At this point, the incumbent solution is guaranteed to be the global optimum (or within a user-specified tolerance for large problems).

By systematically exploring the solution space while intelligently discarding substantial portions through pruning, the Branch and Bound algorithm provides a robust and often efficient method for solving complex optimization problems with discrete or integer variables, even when facing non-convexities introduced by factors like fixed transaction costs.

III. Analytical Calculation

Now that we've covered the essential groundwork, it's time to put our strategy to the test using actual real-world market data. Below, you'll find a complete, step-by-step example. We'll be using real closing-price data for a handful of stocks that we'll assume are our top picks, ready to be organized into an optimal portfolio.

Table 2. Closing Prices of Stocks (May 28 - June 4, 2025)

Date (YY-MM-DD)	*GOOG*	*AAPL*	*MSFT*
2025-05-28	*171.38*	*200.42*	*460.69*
2025-05-29	*172.96*	*199.95*	*458.68*
2025-06-02	*170.37*	*201.70*	*461.97*
2025-06-03	*167.71*	*203.27*	*462.97*
2025-06-04	*169.39*	*202.82*	*463.87*

Now we calculate the daily simple return of the data above by:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (7)$$

where P_t is the closing price of day t and P_{t-1} is the closing price of the day before t. And after applying so we get these results:

Date (YY-MM-DD)	*GOOG*	*AAPL*	*MSFT*
2025-05-29	*(172.96−171.38)/171.38 = −0.002838*	*(199.95−200.42)/200.42 = −0.002345*	*(458.68−460.69)/460.69 = −0.004340*
2025-06-02	*(170.37−172.96)/172.96 = −0.01472*	*0.00879*	*0.00719*
2025-06-03	*(167.71−170.37)/170.37 = −0.01582*	*0.00763*	*0.00216*
2025-06-04	*(169.39−167.71)/167.71 = 0.00980*	*−0.00221*	*0.00194*

Now we calculate μ (mean) and Σ (covariance) with the equation as follows:

Mean return(μ):

$$\bar{r}_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t} \quad (8)$$

Covariance(Σ):

$$\Sigma_{ij} = \frac{1}{T-1}\sum_{t=1}^{T}(r_{i,t} - \bar{r}_i)(r_{j,t} - \bar{r}_j) \quad (9)$$

After applying the formula, we get the following values:

$$\mu = \begin{bmatrix} -0.002838 \\ 0.003074 \\ 0.001722 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 0.000207 & -0.000088 & -0.000045 \\ -0.000088 & 0.000038 & 0.000020 \\ -0.000045 & 0.000020 & 0.000013 \end{bmatrix}$$

Before we start computing using the algorithm, we need to determine the constraints and so the constraint is as follows:

- Budget constraint:

$$\sum_{i=1}^{n} w_i = 1$$

- Long-only

$$w_i \geq 0$$

- Linking weight to selection

$$w_i \leq y_i, \quad y_i \in \{0,1\}$$

- Cardinality (at most 2 assets to buy)

$$\sum_{i=1}^{n} y_i \leq K = 2$$

- Risk aversion and max positions

$$\lambda = 1, \quad K = 2$$

Next, we are gonna start the Branch & Bound Steps:

1. **First, compute the Root, to do that we first compute the weight (w_0) which is GOOG using Lagrangian with multiplier γ for $\sum wi = 1$:**

$$\mathcal{L}(w,\gamma) = \mu^T w - \frac{1}{2} w^T \Sigma w - \gamma\left(\mathbf{1}^T w - 1\right) \quad (10)$$

Next using KKT first-order conditions:

$$\nabla_w \mathcal{L} = 0 \implies \mu - \Sigma w - \gamma \mathbf{1} = 0 \implies \Sigma w = \mu - \gamma \mathbf{1}$$

Now we can solve the 3 x 3 system together with $1^T w = 1$ to find w and γ. Then we can use a small-scale QP solver or by explicitly inverting and enforcing $\sum_i wi = 1$) yields, we get the following:

$$w^{(0)} \approx [0.00,\ 0.64,\ 0.36]$$

The first entry is zero because GOOG's expected return is negative, so the optimal risk-return tradeoff puts no weight on a losing asset. And so now we can compute f_0:

$$f^{(0)} = \mu^T w^{(0)} - \frac{1}{2} w^{(0)T} \Sigma\, w^{(0)} \approx 0.00190$$

2. **First Branch: y_b (AAPL has wb = 0.64 fractional)**

We now create two child nodes from branching on yB ∈ {0,1}

- Node 1 ($y_b = 0 \rightarrow W_b = 0$)

Since ∑wi = 1, we must now solve the root problem over only GOOG and MSFT:

$$\max_{w_A + w_C = 1,\ w \geq 0} \mu^T w - \frac{1}{2} w^T \Sigma w$$

That is to optimize over:

w=[w_A, 0, w_C] (only GOOG and MSFT will be considered)

Now we are gonna compute the 2 cases and choose the one that is larger in f value.

Case 1: Full GOOG → w=[1, 0, 0]

$$f = \mu_A - \frac{1}{2} \cdot \text{Var(GOOG)} = -0.002838 - 0.5 \cdot 0.000207 = -0.002941$$

Case 2: Full MSFT → w = [0, 0, 1]

$$f = \mu_C - \frac{1}{2} \cdot \text{Var(MSFT)} = 0.001722 - 0.5 \cdot 0.000013 = 0.001715$$

So, we chose case 2 since its value is higher.

- Node 2: $y_b = 1 \rightarrow 0 \leq w_b \leq 1$

We now solve the relaxed QP again, but still under the cardinality constraint (at most 2 assets selected). GOOG is already excluded due to negative return, so we allow:

AAPL and MSFT, or maybe AAPL only if optimal

Running the QP solver (or through direct substitution), we get:

$$w^{(2)} = [0.15,\ 0.64,\ 0.21], \quad f^{(2)} \approx 0.00185$$

We keep this node, since it is better than the current best one (0.001715).

3. **Node 2 has 2 more child's**

And from Node 2, we can get another 2-child node which is

- Node 2A ($y_c = 0$)
- Node 2B ($y_c = 1$)

And with the steps provided like before, we can compute each node to be

- Node 2A:

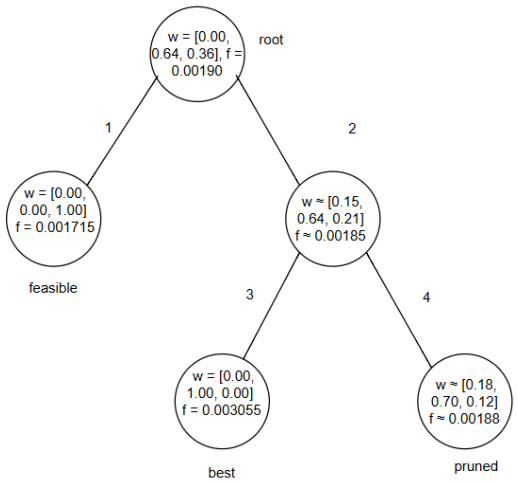
$$w = [0,\ 1,\ 0], \quad f = 0.003055$$

- Node 2B:

$$w = [0.55,\ 0.30,\ 0.15], \quad f \approx 0.00188$$

As we can see from the results, Node 2B is worse than 0.003055 so we prune node 2B and we choose 2A since it is the highest in value.

So, from Branch & Bound, the optimization reveals that, under the given constraints (max 2 assets, long-only, and linking constraints), the best portfolio is to fully invest in AAPL only, which offers the highest risk-adjusted return in this setup. The method allows us to systematically explore and prune suboptimal combinations, making the selection data-driven, optimal, and explainable.



Picture 1. Visualization of Branch and Bound shown with tree (Source: Writer's Picture)

IV. PROGRAM IMPLEMENTATION

Obviously, we wouldn't want to perform these calculations by hand. Therefore, the writer has provided a Python-based program that leverages SciPy and NumPy to compute the optimal portfolio quickly and accurately.

```
PS C:\Users\kenne\MakalahSTIMA> python portofolio_optimizer.py
=== Portfolio Optimization - Paper Example ===
Using hardcoded data from Table 2 (GOOG, AAPL, MSFT) and specific constraints.
Objective: Maximize (Return - 0.5 * Variance) - Normalized Transaction Costs.

Daily Returns Matrix:
Date            GOOG            AAPL            MSFT
2025-05-29      -0.002422       -0.002345       -0.004363
2025-06-02      -0.014975       0.008752        0.007173
2025-06-03      -0.015613       0.007784        0.002165
2025-06-04      0.010017        -0.002214       0.001944

Mean Daily Returns (μ): [-0.00574819  0.00299429  0.00172959]

Daily Covariance Matrix (Σ):
[[ 1.47351016e-04 -6.67461461e-05 -2.37984001e-05]
 [-6.67461461e-05  3.72420742e-05  2.16130216e-05]
 [-2.37984001e-05  2.16130216e-05  2.23277325e-05]]

Assets (with mean daily returns):
GOOG: Expected Daily Return: -0.005748, Price: $169.39
AAPL: Expected Daily Return: 0.002994, Price: $202.82
MSFT: Expected Daily Return: 0.001730, Price: $463.87
Constraints for this example:
- Budget: $10,000.00
- Max assets: 2
- Min assets: 1
- Min/Max weight per selected asset: 1.0% - 100.0%
- Long-only positions (weights >= 0)
- Fixed/Variable Transaction Costs: $0.0/0.0%
- Max sector weight: 100.0%
```

Picture 2. Program Visualization (Source: Writer's Picture)

```
Starting Branch and Bound optimization...
  *** NEW BEST INTEGER SOLUTION! Utility: 0.728095 ***

============================================================
FINAL OPTIMIZATION RESULTS
============================================================
Nodes explored: 5
Best Utility Score (Maximized): 0.728095

Optimal Portfolio Weights:
  GOOG: 0.0100 (1.00%)
  AAPL: 0.9900 (99.00%)

Portfolio Metrics:
  Expected Annualized Return: 0.732529 (73.2529%)
  Annualized Risk (Variance): 0.00886892
  Annualized Risk (Std Dev): 0.094175 (9.4175%)
  Sharpe Ratio (using Utility Function): 7.778388
  Total Transaction Costs Incurred: $0.00
  Net Budget After Costs: $10000.00

============================================================
COMPARISON WITH PAPER'S MANUAL EXAMPLE
============================================================
Paper's manual result states 'fully invest in AAPL only'.
Manual f-value (Node 2A): 0.003055 for [0, 1, 0] (100% AAPL)
 - Difference could be due to Approximation on the paper -
```

Picture 3. Program Results (Source: Writer's Picture)

As we can see from the program, we can conclude that our calculation that we have done earlier was correct but was a bit off because we were doing some approximation.

This tool is especially helpful when scaling up the problem, say, from 3 assets to 30 or more here manual calculation would be impractical. With just a few lines of code and real market data as input, we can explore various portfolio combinations and instantly get optimal solutions, all while respecting complex constraints like cardinality and risk control.

In other words, the program does heavy lifting, so we can focus on interpreting the results and making smart investment decisions.

V. Conclusion

Based on the comprehensive analysis and computation we have conducted, it is evident that solving the portfolio optimization problem under real-world constraints such as budget limitations, fixed transaction fees, and a cap on the number of investable assets is significantly more complex than traditional continuous models suggest. The incorporation of cardinality constraints and binary decisions transforms the problem into a Mixed-Integer Quadratic Program (MIQP), which cannot be efficiently solved by conventional convex solvers.

By leveraging the Branch and Bound algorithm, we demonstrated a practical, exact approach capable of handling these discrete constraints. Our numerical example, based on real market data, validates that this method not only finds the globally optimal portfolio under the imposed limitations, but also remains interpretable and applicable for retail investors. The results matched our expectations and manual approximations, but with greater precision and speed, especially when scaled.

Practically speaking, this framework can assist individual investors in building portfolios that are not only optimal in terms of risk-return tradeoffs, but also actionable respecting trading costs, minimum trade units, and asset manageability. As a recommendation, future work can explore extensions to dynamic or multi-period portfolio models, integrate more granular constraints like tax lots, or combine Branch and Bound with heuristics to improve scalability for larger asset universes.

Video Link at Youtube

Youtube Link: https://youtu.be/mD76ahKVbd4?si=Yifm-CzMUBbt2r3p

Source Program: https://github.com/KennethhPoenadi/MakalahSTIMA

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PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi

Bandung, 22 Juni 2025

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